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KINETIC EQUATIONS FOR THE QUARK CONDENSATE IN THE NJL TYPE MODELS

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In the self-consistent mean field approximation, the Vlasov type kinetic equation is derived within a schematic four-fermion superconducting model of the Nambu — Jona — Losinio type for an arbitrary group of internal symmetry. The Eguchi — Sugawara equation is generalized to the case of finite temperature and density resulting in the Ginzburg — Landau type equation. Perspectives of the implementation of these kinetic equations for describing dynamics of the meson evolution in heavy ion collisions at relativistic energies are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR and at the Saratov State University.

Кинетические уравнения для кваркового конденсата в моделях типа Намбу — Иона — Лазинио

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В рамках схематической 4-фермионной сверхпроводящей модели типа Намбу — Иона — Лазинио для произвольной группы симметрии в приближении самосогласованного поля получено кинетическое уравнение Власова. Получено также уравнение типа Ландау — Гинзбурга, которое является обобщением уравнения Егучи — Сугавары на случай конечных температур и плотностей. Обсуждаются перспективы применения этих уравнений для описания динамики эволюции мезонов при столкновении тяжелых ионов в области релятивистских энергий.

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A microscopic study of the dymanics of heavy-ion collisions is a central problem in the modern relativistic nuclear physics. In the last time much attention has been paid to the derivation of kinetic equations for describing the evolution of hot and compressed hadronic nuclear matter. These results are based mainly on the Walecka model but obtained by using a different technique [1]. A procedure to derive kinetic equations (KE) is not simple

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and, in this connection, the non-equilibrium statistical operator method introduced by D.N.Zubarev [2] has some advantage because it permits one to overcome a large part of the path of deriving KE in a very general form without any specification of the interacting system. Recently, the relativistic-invariant generalization of the Zubarev method has been carried out in paper [3], where within perturbation theory the quantum KE with collision integrals of the first and second order in the interaction coupling were obtained. In particular the KE for the baryonic sector of the Walecka model were considered and the collision integrals of the first (Vlasov's type) and second (Bloch's type) order were investigated in the slowly changing field approximation. Spin effects have been involved into consideration naturally. These results can be easily generalized to other kinds of interaction with vertices of the 'three-tails" type and it has been done in [3] for the case of chiral symmetry theory.

In this paper we shall take the method of ref.[3] as a starting point for deriving the KE of Vlasov's type for the quark condensate within the Nambu — Jona — Losinio (NJL) type models at finite temperature and density. As the first step, we consider the simplest version of the NJL model with the only kind of fermions and with chiral and translational symmetry breaking, the so-called Eguchi — Sugawara version of the NJL model [4]. The more realistic quark SU(N) NJL model which may serve as an approximation of QCD in the long-wave length limit will be discussed in other paper. It is noteworthy that in the original paper [5], the translation symmetry theory at zero temperature has only been considered. The extension of the NJL model to the case of non-zero temperature can be find in papers [6]—[9]. The Eguchi — Sugawara model [4] takes into account both chiral and translational symmetry breaking but at zero temperature. In the present paper this version of the NJL type models will be extended to temperatures different from zero.

The initial Lagrangian density of the NJL type model with two kinds of the coupling constants has the following form [4]:

$$(x) = \bar{\psi} i\gamma\nabla\psi + g\,[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] - g'\,[(\bar{\psi}\gamma_n\psi)^2 + (\bar{\psi}\gamma_5\gamma_n\psi)^2].$$

In the Hartree — Fock approximation, we get from here the Eguchi — Sugawara Lagrangian density

$$_{ES}(x) = \bar{\psi}(x)\,[i\gamma - \hat{m}(x)]\psi, \qquad (1)$$

where

$$\hat{m}(x) = m^S(x)I + i\gamma_5 m^P(x) + \gamma^n m_n^P(x) + \gamma^n m_n^V(x) + \gamma_5\gamma^n m_n^A(x),$$
$$m^S(x) = -2g\, Sp\, \langle \overline{\psi}(x)\psi(x) \rangle_\tau ,$$
$$m^P(x) = -2ig\, Sp\, \gamma_5 \langle \overline{\psi}(x)\psi(x) \rangle_\tau ,$$
$$m_n^V(x) = (4g' + g)\, Sp\, \gamma_n \langle \overline{\psi}(x)\psi(x) \rangle_\tau ,$$
$$m_n^A(x) = (4g' - g)\, Sp\, \gamma_5\gamma_n \langle \overline{\psi}(x)\psi(x) \rangle_\tau . \tag{2}$$

Unlike ref. [4], the averaging in these expressions is performed by using the non-equilibrium statistical operator $\rho(\tau)$, i.e. $\langle \dots \rangle_\tau = Sp \dots \rho(\tau)$. This operator introduces the local temperature $T(x)$ in this model. Equations (2) define a set of the non-equilibrium order parameters of the theory which are considered as a measure of the chiral symmetry breaking.

Formally, the Lagrangian density (1) corresponds to the quasi-free motion of a particle with the translation non-invariant «mass» matrix $\hat{m}(x)$. According to eq. (1), the quadrical equation of motion is given by ($f_{,n} \equiv \partial f(x)/\partial x^n$)

$$\{\nabla^2 - i\gamma^n \hat{m}_{,n}(x) - [\gamma^n, \hat{m}(x)]\,\nabla_n - \hat{m}^2(x)\}\, \psi(x) = 0. \tag{3}$$

Following ref. [3] and taking into account this quasi-free motion equation, the general KE can be written down in the approximation (2) as follows

$$p^n \frac{\partial}{\partial x^n} f_{\alpha\beta}(x, p) = -i\sqrt{p^2}\,(2\pi)^{-4} \int d^4y\, e^{-ipy} \times$$
$$\times \langle [\overline{\psi}_\beta (x + y/2)\, \psi_\alpha (x - y/2),\, H_{ES}] \rangle_\tau . \tag{4}$$

Here the Hamiltonian H_{ES} corresponds to the Lagrangian density (1) for the Eguchi — Sugawara model and $f_{\alpha\beta}(x, p)$ is the relativistic Wigner function

$$f_{\alpha\beta}(x, p) = (2\pi)^{-4} \int d^4y\, e^{-ipy} \langle \overline{\psi}_\beta (x + y/2)\, \psi_\alpha (x - y/2) \rangle_\tau . \tag{5}$$

Keeping the first order terms in coupling counstants in eq. (4), one can get the resulting KE of the Vlasov type in the Wigner representation, to be nonlocal equation for (5). In the long-wave approximation, it reads:

$$p^n \frac{\partial f}{\partial x^n} = \frac{1}{4} \{f, \{\hat{m}_{,n}, \gamma^n\}\} + \frac{i}{2} \{f, [\hat{m}, p\gamma]\} - \frac{1}{4} [\frac{\partial f}{\partial p_n}, [\frac{\partial \hat{m}}{\partial x^n}, p\gamma]] -$$

$$- \frac{1}{4} \{\frac{\partial f}{\partial p_n}, \frac{\partial \hat{m}^2}{\partial x^n}\} + \frac{1}{4} [\frac{\partial f}{\partial x^n}, [\hat{m}, \gamma^n]] + \frac{i}{2} [f, \hat{m}^2]. \quad (6)$$

Here the order parameters (2) can be expressed through the Wigner function (5)

$$m_a(x) = Sp\, \Gamma_\alpha \int d^4p\, f(x, p), \quad (7)$$

where the index enumerates the order parameters from the set (2). The matrix Γ^α consists of a combination of corresponding coupling constant and γ-matrices. So, the correlations (7) mean that the KE (6) is a system of nonlinear integro-differential equations for the Wigner function and its zero moments.

Discussing the final KE (6), one should note that the system under consideration will be translationally invariant at thermodynamical equilibrium. This leads to the following requirement for the equilibrium Wigner function $f^{(0)}$:

$$[f^{(0)}, \hat{m}^2_{(0)}] + \{f^{(0)}, [\hat{m}_{(0)}, p\gamma]\} = 0. \quad (8)$$

The matrix structure of the equilibrium order parameter is defined in ref. [9]: $\hat{m}_{(0)} = m_T I + \delta\mu\gamma^0$, where $\delta\mu$ is the chemical potential correction and m_T is the scalar mass at finite temperature. We have $m_T \to m_\infty$ at $T \to 0$, where m_∞ is the mass of a condensate boson in the NJL model at $T = 0$. The last relation for the $\hat{m}_{(0)}$ agrees with a general structure of the Wigner $f^{(0)} = aI + b_n\gamma^n$ function [10]. After substitution of $\hat{m}_{(0)}$ and $f^{(0)}$ into eq. (8) we get the equality $a = b$ assuming $b_n = b(p)p_n$. Below we shall consider a simpler case when there is no vector fields, which results in the zero changing of the chemical potential in the equilibrium state, $\delta\mu = 0$.

In this case it is possible to derive a closed system of equations for weakly non-equilibrium order parameters $\delta\hat{m} = \hat{m} - \hat{m}_{(0)}$. Let us assume that $\delta f(x, p) = f(x, p) - f^{(0)}(p)$ is also small and substitute $\delta\hat{m}$ and $\delta f(x, p)$ into eq. (6) restricting ourselves by linear terms for small deviations from an equilibrium state. After carrying out the Fourier transformation, we arrive at the following system of equations for non-equilibrium order parameters:

$$\delta m_a(k) = \frac{1}{2} Sp \int d^4p \frac{\Gamma_a}{pk} (\frac{1}{2} \{f^{(0)}(p), \{\delta\hat{m}(k), k\gamma\}\} -$$

$$- \{f^{(0)}(p), [\delta\hat{m}(k), p\gamma]\} - \frac{1}{2} [k \frac{\partial f^{(0)}(p)}{\partial p}, [\delta\hat{m}(k), p\gamma]] -$$

$$- m_T \{k \frac{\partial f^{(0)}(p)}{\partial p}, \partial\hat{m}(k)\} - 2m_T [\partial\hat{m}, f^{(0)}(p)]). \tag{9}$$

These equations can be considered as a certain analogy of the gap equations for the non-equilibrium state.

The initial model Lagrangian is independent of internal symmetry of fields involved. So, the KE (6) and (9) based on this Lagrangian are valid for any SU(N) version of the NJL model. In this sense the above-derived KE (6) and (9) can be considered as generalized KE. In particular, the vector meson degrees of freedom in (x) may be eliminated by the Fierz transformation.

The Vlasov type KE (6) is reasonable in the self-consistent field approximation. It is not very hard to get the collision integral of the Bloch type to be second order in the coupling constant (see ref.[3]). It is noteworthy that in this case a quasiparticle scattering should be expressed in terms of «residual» interaction, i.e. for the Lagrangian $_{in} = {}_{in} - \bar{\psi}\hat{m}\psi$. By constructing $\hat{m}$, we have $\langle \bar{}_{in} \rangle_{in}$ in the Hartree — Fock approximation.

In this sense, a free motion is not yet a foundation of the perturbation theory in the NJL type models. However, it does be so when considered with the Lagrangian density (1). In this case the mass shell equation is given by $p^2 = m_T^2$ and depends on $T(x)$.

Eq.(6) describes the kinetic stage of the system evolution when a fast motion is eliminated from consideration. However, the exact equation of motion for the Wigner function (5) can be derived by means of the method of ref.[11]. In the Hartree — Fock approximation for (1), it results in the following equation:

$$\gamma^n (p_n + \frac{i}{2} \frac{\partial}{\partial x^n}) f(x, p) = \exp (- \frac{1}{2} \frac{\partial}{\partial x^n_{(m)}} \frac{\partial}{\partial p_n}) \hat{m}(x) f(x, p). \tag{10}$$

The sign (m) means that the operator $\partial/\partial x^n_{(m)}$ acts only on the matrix $\hat{m}(x)$. It is of interest that eq.(10) may serve as a basis for deriving a closed system of wave equations in fields (7). These equations are a straightforward generalization of the Eguchi — Sugawara wave equations for the case of finite temperature. In the limiting case $\mu \to 0$ and $T \to 0$ we have $\mu_T \to \mu_\infty$ and our equation of the Ginzburg — Landau type is reduced to that of ref.[4].

Eqs.(6), (9) and (10) may be used for extracting the information of various kinds about a meson subsystem of nuclear matter. So, their solution for the Wigner function can provide the canonical kinetic description of «bozonized» quark systems (as well as hydrodynamical description, problems of the proper frequency and decrement constants etc.). Particularity of this description is the presence of the order parameters $\hat{m}(x)$. In some cases, it is possible to write down a closed system of equations for bosonic degrees of freedom without any reference to a fermionic subsystem. Therefore, eqs.(6), (9) and (10) seem to be quite promising for the description of dynamical phase transition of the chiral symmetry restoration in a bosonic subsystem and for the study of meson properties in hot and dense nuclear matter.

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